Temperature as a bifurcation parameter in nonlinear electronic circuits

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It is shown that temperature variations can bring about a series of bifurcations in the behavior of a nonlinear electronic circuit. For the semiconductor diode used in the experiments the increase of temperature above room temperature yields the period-doubling route to chaos, periodic windows, and a return to the ordered state. The most striking finding is that over the temperature range which is of general interest (i.e., just above room temperature) the temperature behaves as a genuine external control parameter of the system. We explain our observations by suggesting that the temperature is a scaling parameter of the applied voltage.

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Probably the simplest system in which bifurcation phenomena and chaotic behavior can be followed by well controlled experiments is the nonlinear electronic circuit. Indeed, it was a short time after Feigenbaum's explanation of the period-doubling route to chaos [1] that Linsay [2] found it in a circuit composed of a resistor R, an inductor L, and a nonlinear element which was a semiconductor diode with a voltage-dependence capacitance and conductance. In the many experiments [3–6] that followed his work the circuit "control," or "drive," parameter was the rms value V_{ac} or the peak-to-peak voltage value $V \equiv \sqrt{2} V_{ac}$ of the sinusoidal voltage applied to the circuit. More recently, the role of the frequency of the applied ac signal, f, has been studied [7,8] and corresponding phase diagrams in the parameter space have been determined [7,8]. Of course, the particular behavior of the circuit in general, and the bifurcation sequence in particular, depend also on the V-independent values of R and L, as well as on the capacitance-voltage (C-V) and current-voltage (I-V) characteristics of the nonlinear element [2,8].

The purpose of the present paper is to point out that the variation of the ambient temperature T of the nonlinear element is playing the same role as the variation of V_{ac}. We further note that while the variation in the temperature amounts simply to variation in the characteristics of the diode this variation is the *only* continuous and controllable variation that can be induced from outside the circuit. This makes the circuit temperature a *special* bifurcation control parameter. The above observation has two consequences. From the basic physics point of view it raises the question of how do the changes in the diode's nonlinear characteristics determine the bifurcation sequence of the circuit in general, and how does the temperature affect this sequence in particular? From the more practical point of view, we note especially that, unlike V_{ac} or f, one is not always in control of T. Hence the possibility that an electronic circuit will get into a chaotic oscillation mode, due to the rise of temperature in the temperature range which is normal for electronic circuits operation, should be of great concern to electronic circuit-board planners. The understanding derived in this paper is expected to help in the evaluation and/or the control of such effects.

The circuit studied in this work, which is shown schematically in Fig. 1, as well as the procedure of the measurement, were much like those originally applied by Linsay [2]. We have used, however, the power diode 1N4998 instead of a varactor diode and operated the circuit at a much lower frequency to fit the range of our Hewlett Packard 3582A spectrum analyzer. The capacitance voltage (C-V) and current voltage (I-V) characteristics of such diodes are well understood [9] and are easily measured [10]. They are well known to be weakly dependent on temperature, in the reverse bias regime, and strongly dependent on temperature, in the forward bias regime.

Turning to the study of the bifurcation series, we have followed the nonlinear response of the RL-diode circuit by measuring the power spectra of the voltage across the diode. This was done first in the conventional mode of operation, i.e., as a function of the ac drive voltage V_{ac}. The results were very similar to those reported by Linsay [2] (see also below). In our measurements of the power

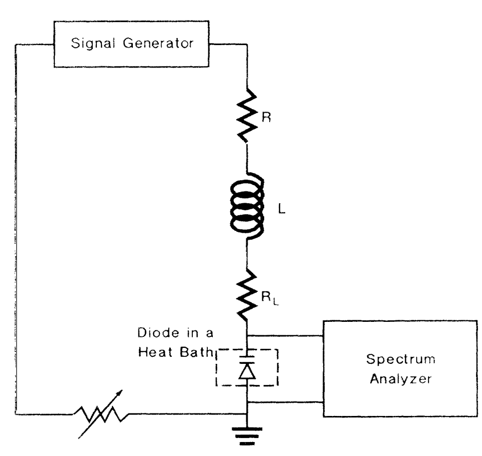

FIG. 1. A schematic description of the circuit used in the measurement.

spectrum as a function of the ambient temperature T, the diode was immersed in a large oil bath and the temperature was monitored by a thermometer that was adjacent to the diode. In this preliminary study we did not make an effort to determine the temperature with a higher accuracy than 0.25 °C, since this corresponded to our ability to determine the onset of the various bifurcations on the spectrum analyzer. The value of V_{ac}, its frequency f, and the variable circuit resistor were chosen so that we obtained as many bifurcations as possible in the temperature range studied. The latter range, i.e., $4 \leq T \leq 160$ °C, was chosen since it is convenient experimentally and since it is relevant to applications under normal conditions. We have taken then the values $V_{ac} = 0.99$ V and $f = 15$ kHz, when the passive parameters of the circuit were $R = 50\Omega$, $R_L = 33\Omega$, and $L = 81.6$ mH. The variable resistor was set at zero. Hence, in the results obtained, the 15-kHz peak is simply the fundamental spectral peak and the peak at $f = 0$ is the Fourier transform "reflection" of this peak [11].

The evolution of bifurcations as recorded in the above temperature range is shown in Fig. 2. In Fig. 2(a) one observes a tiny reminiscence of the first bifurcation peak in order to mark the end of the simple periodic ("ordered" or "linear") behavior of the circuit. The well developed (or saturated [2]) first pitchfork bifurcation is shown in Fig. 2(b). This bifurcation is followed by the second [Fig. 2(c)], the third [Fig. 2(d)], and the fourth [Fig. 2(e)] pitchfork bifurcations. The behavior shown in Figs. 2(b)–2(e) is in agreement with the pitchfork bifurcation sequence observed by Linsay [2] and by Testa, Perez, and Jeffries [3], *except,* of course, for the fact that the bifurcation sequence is controlled here by the temperature rather than by the applied ac voltage.

At higher temperatures, in Fig. 2(f), we see the onset of chaotic (or a continuous spectrum) behavior. The structure observed in the continuous power spectrum reflects the nonuniform probability of finding a given frequency in the corresponding chaotic regime. Indeed, the higher amplitudes of the relatively wide peaks correspond to the darker "vale regimes" in the bifurcation maps [1,11]. Above the two-band chaotic regime which we call Ch_2 [Fig. 2(f)] we find a variation in the continuous noise character. We denote this chaotic regime Ch_1 [Fig. 2(g)]. The chaotic regime Ch_1 is followed then by period tripling. This feature, which is shown in Fig. 2(h), is just the period-3 window found, beyond the chaotic regime, in many [4–8] (but not all [3]) RL-diode circuits. Note that the equal amplitudes of the window peaks are in sharp contrast to the rescaled amplitudes which characterize the period-doubling route to chaos (see above). A further increase of the temperature drives the circuit back to its "ordered" periodic mode by reverse bifurcation. This is reflected by the period doubling seen in Fig. 2(i), which is followed by the periodic behavior seen in Fig. 2(j). We did not find any more bifurcations up to $T = 160$ °C, which was the highest temperature applied in this set of experiments. The latter observation is not too common in the bifurcation maps of RL-diode circuits when V_{ac} is the control parameter, but it has been observed [7]. However, as far as we know, this behavior has not been discussed previously (see below). Following the above results, we can conclude then that the temperature acts as the drive voltage, i.e., as a "genuine-independent" control parameter of RL-semiconductor diode circuits.

Following the above behavior, the question that arises is, which of the better known theoretical predictions can be tested if we assume that the temperature is a genuine-independent control parameter? Naturally the first prediction that comes to mind is the universal bifurcation

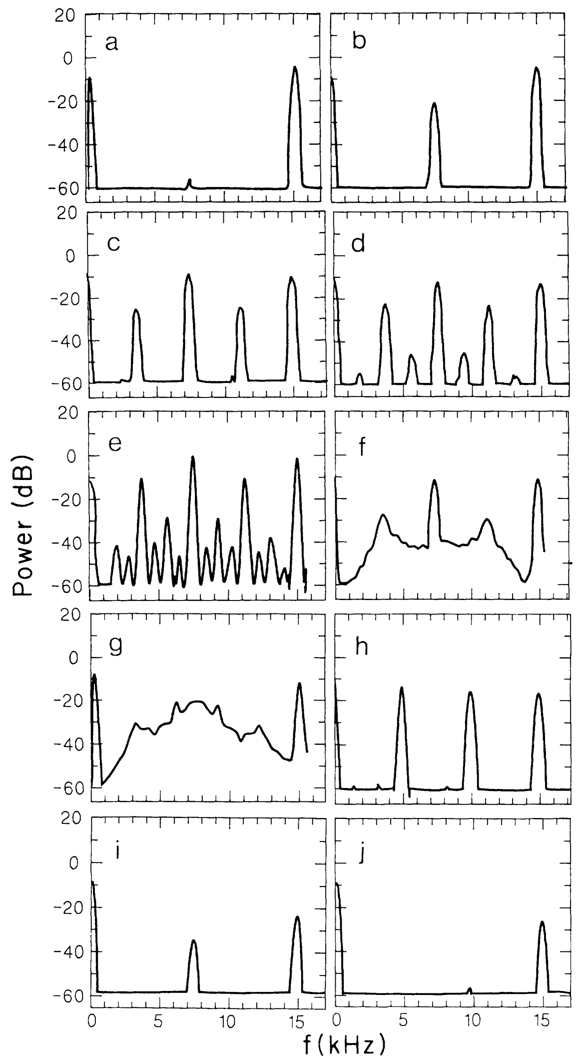

FIG. 2. Power spectra measured in the circuit shown in Fig. 1 using the diode 1N4998. The applied sinusoidal voltage had an amplitude of $V_{ac} = 0.99$ V, adjusted so that with $f = 15$ kHz, the circuit is at the threshold of the first bifurcation at $T_1 = 4$ °C (a). The first clear pitchfork bifurcation appears at $T_2 = 7$ °C (b), the second pitchfork bifurcation is clear at $T_3 = 98$ °C (c), the third pitchfork bifurcation is clear at $T_4 = 108$ °C (d), and the fourth pitchfork bifurcation is found at 109 °C (e). With a further increase of temperature, e.g., at $T_5 = 110$ °C, a continuous two-band chaos takes over (f), and then it changes its character at $T = 118$ °C (g). Beyond the chaotic regime, at $T = 132$ °C, one finds a period-3 window (h), which is followed by period doubling at 156 °C (i), and then a return to the periodic behavior at $T = 160$ °C (j).

convergence coefficient, which is obtained by measuring the increments in the control parameter between subsequent bifurcations [1,11]. In Feigenbaum's universal theory [1] for the one-dimensional map this value for the higher bifurcations is $\delta=4.669$. From Figs. 2(b)–2(d) we see that $(T_3-T_2)/(T_4-T_3)=9\pm1$ and from Figs. 2(c)–2(e) we see that $(T_4-T_3)/(T_5-T_4)=5\pm1$. As we argue below, these results are in excellent agreement with the universal predictions. Another agreement with the Feigenbaum theory is tested by turning to the other prediction of Feigenbaum's theory, i.e., the universal rescaling factor α. This is done here by considering the rescaling exponent of the power amplitude in consequent bifurcations μ, which is simply related [1,11] to α. Feigenbaum's prediction [1,2] for μ is that the ratio of subsequent period-doubling power-spectrum peaks is 8.2 dB. As is clearly seen in Figs. 2(b) and 2(c) the experimental ratio is 9 ± 1 dB. The agreement of the above results with the Feigenbaum predictions is an indication that in the corresponding temperature range the temperature can be considered to be a simple control parameter of a one-dimensional map.

Looking for an explanation for the above-described behavior, we have considered the simplest model suggested thus far to account for a voltage-driven sequence of bifurcations in *RL*-diode circuits. This model consists of a two-segment piecewise-linear capacitor [6]. In this model the nonlinear element is a diode which has a low constant-value capacitance C_1', under an application of a bias that is lower than a certain threshold E_0, and a high constant-value capacitance C_2', under an application of a higher bias. In this model the diode has an infinite resistance and thus the nonlinear behavior is solely determined by this capacitor. For our purpose the most important conclusion of this simple model (which was analyzed analytically [6]) is that the drive parameter in such a nonlinear circuit is V/E_0 rather than V.

Turning to the real nonlinear element used in our experiments, we have noted, as have previous researchers [5,7] that by displaying the voltage drop across the diode (in the circuit shown in Fig. 1) on an oscilloscope, one finds that under the forward bias portion of the cycle this signal has a wide flat maximum at a voltage value V_f. Considering the *C-V* characteristic of the diode, we know that V_f corresponds to a certain value of the forward bias capacitance C_2. In our experiments we found that, while V_f depends on the ambient temperature, the corresponding capacitance associated with V_f is essentially independent of temperature. Its value, as determined from the *C-V* characteristics was found to be about $2.5\times10^{-8}F$. We thus suggest that to a first approximation the real nonlinear element in the circuit studied can be described by the above two-segment piecewise-linear capacitor model, where C_2 is the constant capacitance which is obtained at the forward saturation value V_f. We identify then the C_2 value of the real diode with an effective C_2' value (see above). Consequently, we suggest that the main effect of the temperature variation on the diodes' operation in the circuit is the variation of V_f. In our circuit, over the temperature range under study ($4\le T\le160\,^{\circ}\mathrm{C}$), the measured dependence of V_f on T is well approximated by

$$V_f=0.51-0.0029T\ , \tag{1}$$

where V_f is given in volts and T in °C.

Following the above comparison between the real diode behavior and the simple model behavior, it is quite natural to identify the observed value of V_f with the value of E_0 in the simplified model [6] described above. This point of view is strongly supported by the observation that one can use V/V_f as a "reduced drive parameter" [5] as well as by the behavior observed in Figs. 2(i) and 2(j). This latter finding of the return of the circuit to an ordered behavior at high temperatures, combined with Eq. (1), indicates that this behavior is associated with the V_f approach to a zero value. Indeed, we have confirmed that this is the case by computer simulations [12] of various *RL*-diode circuit models that show such a behavior when V_f or E_0 approach a zero value. In our present study the increase of temperature [as can be seen in Eq. (1)] amounts to the decrease of V_f, which we interpret as the decrease of E_0.

Identifying E_0 with V_f, we are now in a position to explain the observed series of temperature-driven bifurcations. Let V_n denote the threshold voltage for the nth pitchfork bifurcation when E_0 is fixed and let E_0^n be the threshold value for the nth bifurcation when V is fixed. The assumption that the circuit control parameter is V/E_0 yields then that

$$(V_n-V_{n-1})/(V_{n-1}-V_{n-2}) =[(E_0^n-E_0^{n-1})/(E_0^{n-1}-E_0^{n-2})](E_0^{n-2}/E_0^n)\ . \tag{2}$$

Using Eq. (1) and our identification of $V_f=E_0$ we may thus conclude that

$$(V_n-V_{n-1})/(V_{n-1}-V_{n-2}) =(T_n-T_{n-1})/(T_{n-1}-T_{n-2})(E_0^{n-2}/E_0^n)\ . \tag{3}$$

Hence if the ratio on the left-hand side approaches the Feigenbaum δ, the ratio on the right-hand side should also do so, but the convergence will be slower. In order to check this argument, and in order to map the general behavior of the circuit, for the effect of the as yet unstudied but important temperature parameter, we have measured the *V-T* phase diagram. The results are shown in Fig. 3. This phase diagram has been obtained by increasing V_{ac} at fixed T values, in variation with the fixed-V_{ac}-variable-T sequence studied above (see Fig. 2). We note that for the higher bifurcations (above the period-3 window) our map, like the *V-f* map [5,7], shows hysteresis effects such as those to be expected from the fact that the system has multiple basins of attraction [4,11]. Qualitatively, the sequence of bifurcations and the "paraboliclike" shapes of the boundaries between the different "bifurcation phases" resemble the results obtained for the *V-f* diagrams [7]. The boundaries beyond the chaotic regime, and their dependence on the initial conditions, will be discussed elsewhere [12]. For the justification of our

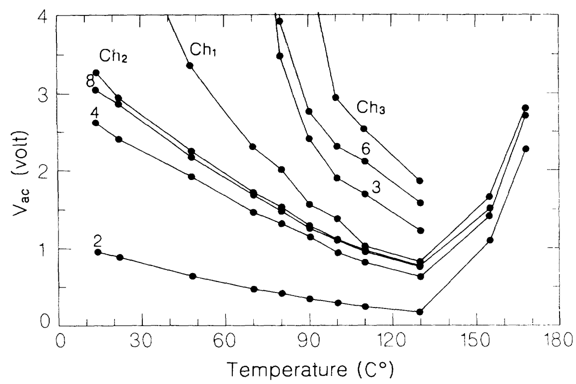

FIG. 3. An experimental phase diagram in parameter space, for the circuit shown in Fig. 1, using diode 1N4998. The various notations indicate the phase-space region of a particular bifurcation. Hence, the 2, 4, and 8 numbers indicate the order of the pitchfork bifurcation, the 3 and 6 numbers indicate the periods in the corresponding windows, while Ch_2, Ch_1, and Ch_3 are different chaotic states that are characterized here by different power spectra [see, e.g., Figs. 2(f) and 2(g)].

interpretation of the observed behavior, the most important observation concerning this figure is the confirmation of the prediction given by Eq. (3). Indeed, following the V_n series for a constant temperature we find that $(V_2-V_1)/(V_3-V_2)$ and $(V_3-V_2)/(V_4-V_3)$ for a given temperature are of the order of 5, i.e., very close to the Feigenbaum $\delta=4.699$. On the other hand, we find (see also Fig. 2 above) that for a given V_{ac} the value of $(T_2-T_1)/(T_3-T_2)$ is of the order of 9, while the value of $(T_3-T_2)/(T_4-T_3)$ is of the order of 5, in good agreement with the predictions that follow Eqs. (1) and (3), if we use our $V_f=E_0$ identification. Hence, our crude determination of the above ratios lends strong support to our suggestion that the temperature (via V_f) is a voltage-scaling parameter. In view of the above observations we speculate that the E_0 parameter (the precise definition of which may vary for various nonlinear circuits) is the principal voltage-scaling parameter that determines the bifurcation series in nonlinear electronic circuits. However, more work is needed to confirm or disprove the generality of this suggestion.

In conclusion, the effect of temperature on the diode characteristics appears to make this parameter a genuine-independent control parameter in nonlinear circuits. This parameter appears to have the role of rescaling the circuit drive voltage in the temperature range, which is very important for practical applications.

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